**Title: When and Where Will It Stop?**

**Brief Overview:**

This learning unit provides two activities to investigate empirically how long (in both time and distance) it takes a vehicle to brake to a stop in relationship to its velocity. Activity #1 is suitable for all high school students, while Activity #2 is suitable for students with a somewhat higher level of mathematical maturity (typically pre-calculus or above). Analysis and data collection will involve the TI-83 and the CBR motion detector.

**Links to NCTM 2000 Standards:**

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

- **Patterns, Functions, and Algebra**
  Students will investigate empirically the relationship between velocity and braking time, as well as the relationship between velocity and braking distance. They will use their intuition, with the aid of graphing calculators, to speculate upon, graph, and refine their understanding of these relationships.

- **Measurement**
  Students will use a sonar ranging device and suitable software on their graphing calculators to measure time, distance, and velocity data points for each trial.

- **Data Analysis, Statistics, and Probability**
  As they gather the data, students will answer questions about the shapes of the graphs produced by the commercial sonar-ranging software. After gathering all the data, students will create summary scatter plots, will answer questions about the shapes of the scatter plots, and, using instructions provided in the activities, will produce suitable regression equations to attempt to fit the data. Students will then use the regression equations to predict future behavior of the vehicle.

**Links to National Science Education Standards:**

- **Unifying Concepts and Processes**
  This learning unit uses the empirical method, graphing (independent vs. dependent variables), and regression as unifying concepts and processes.

**Grade/Level:**

Activity #1: Grades 9-12
Activity #2: Grades 11-12 (Pre-calculus recommended)

**Duration/Length:**

2 to 4 days

**Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Using the TI-83 or TI-83 PLUS graphing calculator
- The concept of rate, distance, and time
- The ability to embrace new definitions (e.g., acceleration and deceleration) and unfamiliar techniques, provided they are presented in sufficient detail

**Objectives:**

Students will:

- gain experience in investigating quantitative data using empirical techniques.
- learn how to use technology to gather time, distance, and velocity data.
- see how to produce and analyze scatter plots and regression functions.
- explore the relationships between velocity and braking time and distance.

**Materials/Resources/Printed Materials:**

- At least one TI-83 or TI-83 PLUS calculator
- At least one Texas Instruments Calculator-Based Ranger ("CBR"), clamp, and cable
- Medium-sized toy car or wheeled physics vehicle
- Metal or wooden ramp with smooth transition at end
- Level runout area (a carpeted floor is ideal)
- Pan balance or beam balance (optional)
- Weights (optional)
- Printed materials: Activity #1, Activity #2, and Teacher's Guide

**Development/Procedures:**

Note: For convenience, these procedures are repeated, with some additional details, in the Teacher's Guide.

For best use of time, the apparatus should probably be set up in advance as shown in the Teacher's Guide. Although students would learn a great deal by designing and setting up their own ramps and runout areas, doing this may take too much time.

Regardless of the activity chosen (#1 or #2), begin by introducing the unit and relating it to the real world. Most students, especially sophomores and later, are very concerned with driving and are very interested in the distance needed to brake to a stop, and possibly also the time required. Explain that we will begin by looking at the time needed to come to a stop, and later we will examine the distance required.

Organize students into groups if you have multiple calculators and/or CBRs. (Otherwise, choose students in rotation to be data collectors for the entire class.) Then proceed to the chosen activity (#1 or #2), and let students work at their own pace. It is important to monitor the early data collection to make sure that students are getting curves of reasonable shape. If the curves are too bumpy, remind them to turn smoothing on ("light" should be sufficient) as shown in the CBR setup instructions on their worksheets.

At the end, or at least after a majority of the students have managed to produce a predictor function, reassemble the students for a contest to guess a trial of velocity that has not been charted before. Have the students predict where the car will stop, and allow them to use any of the methods that they have discovered. Different methods are reasonable, and different methods can produce correct results.

**Assessment:**

For each activity, numbered questions walk the students through each step and provide opportunities for them to reflect upon what they have learned up to that point. Weighting of point values is left to the discretion of the teacher. In many cases, a single completion grade may be appropriate, with a bonus for students or groups who achieve accurate predictions in the final contest.

**Extension/Follow Up:**

The Teacher's Guide includes a number of engineering-type questions to pose to students, as well as two suggestions for extending the experiment, namely (1) computing the value of the gravitational constant $g$ and (2) investigating the effect of vehicle mass on stopping distance.

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Name:______________________________ Date:_____________ Period:___________

# Lab Activity #1:
# When Will It Stop, Where Will It End?

## I. Objective:

In this lab activity, you will be studying the relationships between a moving vehicle's velocity and its stopping time and stopping distance.

## II. Materials Needed:

CBR
TI-83 Graphing Calculator
Ramp
Model Car

## III. Introduction:

In this activity you will investigate the relationship between the velocity of a moving car and how well the car is able to stop. To consider how well a car stops we must look at two factors—how much time it takes for the car to stop and how much distance it takes for the car to stop. To explore these relationships we will simulate the scenario of a car putting on the brakes by running our model car over a carpeted surface. Much as the brakes of a car slow it down by applying a frictional force, the carpet will slow down our model car.

Like all great scientists, we will begin our experiment by making predictions of what is going to happen.

1. Will a faster-moving car take more time or less time to stop than a slower-moving car?

   ___________________

2. Will a faster-moving car require more distance or less distance to stop than a slower-moving car?

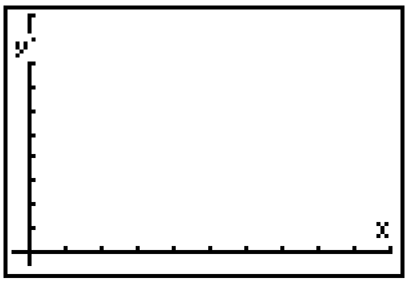
   ___________________

You will be collecting data using the CBR and graphing the results on the TI-83 in order to examine two relationships – velocity vs. stopping time and velocity vs. stopping distance. You will use the ramp to give the model car an initial velocity ($v_0$). Starting the car from different points on the ramp will, of course, give the car different initial velocities. Since the initial velocity is the variable in the experiment that we are going to control, it is our independent variable, which we will always put on the *x*-axis. The *y*-axis will be used for the dependent variable (either stopping time or stopping distance).

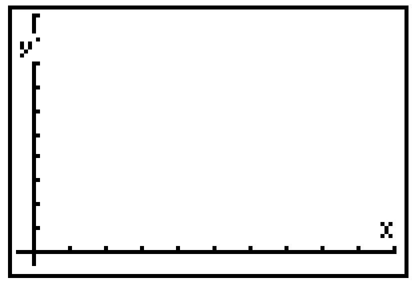
What do you think the two graphs will look like when we are all done? Do not worry so much about exactly how much time or distance will be needed. Instead, think about whether the graphs should increase or decrease, and take a guess about what the shape of the graphs will look like in general.

3. Record your guesses in the boxes below.

Stopping Time (*y*) as a function of Velocity (*x*)



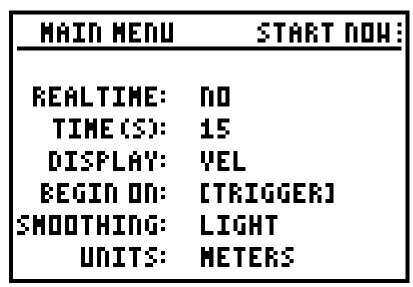

Stopping Distance (*y*) as a function of Velocity (*x*)



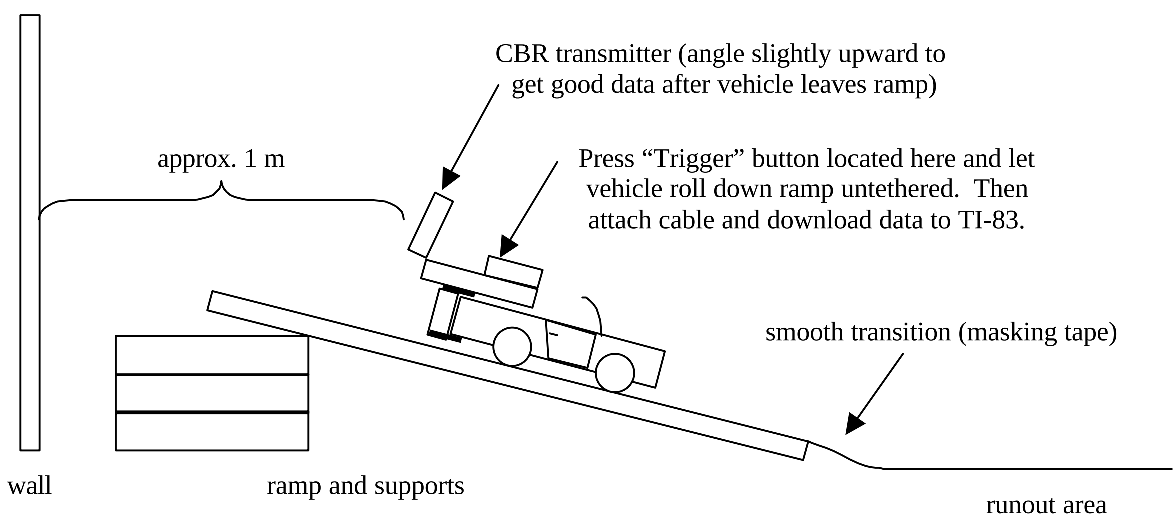

Let’s find out how good these predictions are!

## IV. Setting Up:

- Your teacher will describe how to run the RANGER program and change the settings. When you have finished adjusting the settings, the calculator screen should look like this:




- Check with your teacher to make sure that your ramp matches the diagram below. Clear a path for your model car.



- Make sure that the CBR is mounted securely to the top of the model car with the sensor pointed at the wall behind the ramp. **Make sure to angle the CBR sensor up a little bit so that it does not pick up the ramp as it rolls away.**

- At this point the CBR should not be connected to the calculator by the cable. After each run of the car, you will connect the cable and retrieve the data.

## V. Procedure:

- Press the trigger button on the CBR and release the car from any point on the ramp. Let the car come to a complete stop on its own. **If the car hits an obstacle, the data will not be usable!** Move your ramp if you need to.

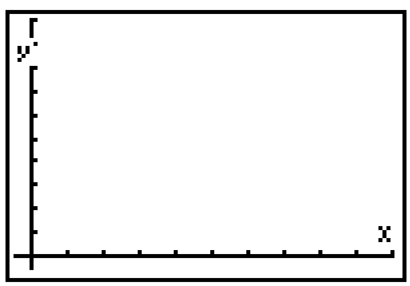
- Connect the CBR to the calculator using the provided cable.

- Execute the RANGER program. (On the TI-83, press **PRGM** as needed to get a blank screen; then highlight RANGER, and press **ENTER** twice.)

- Press the **5** key to select the "TOOLS" option, and then press **1** to get the data from the CBR.

4. The RANGER program will automatically display a graph with time as the independent variable and velocity as the dependent variable. Draw the display in the space below.

Velocity (*y*) as a function of Time (*x*)




5. Draw three points on the graph and label them with the letters A, B, and C to show when each of these things happened:
   A. The car was released on the ramp
   B. The car left the ramp and went onto the carpet
   C. The car came to a stop

6. Should the car gain velocity or lose velocity as it goes down the ramp?____________

7. Does the graph increase or decrease as the car goes down the ramp?______________

8. Should the car gain velocity or lose velocity as it rolls on the carpet?______________

9. Does the graph increase or decrease as the car rolls on the carpet?________________

Press **TRACE** and use the ← and → keys to trace the graph to answer questions 10 through 16.

10. At what time did the car leave the ramp and go onto the carpet? (We will call this value the initial time or $t_o$.) __________________

11. What was the velocity at time $t_o$? (We will call this value the initial velocity or $v_o$.) __________________

12. At what time did the car come to a stop? (We will call this the final time or $t_f$.) _________________

13. What was the velocity at time $t_f$? (We will call this value the final velocity or $v_f$.) __________________

14. On what interval of the domain is the graph decreasing? _______________________

15. On what interval of the domain is the graph increasing? _______________________

16. What is the range of the function? ________________________

The CBR automatically records distance information as well as the time and velocity information displayed on the graph. You will now look at a table of these values to answer questions 17 and 18.

- First, you must exit the RANGER program by pressing **ENTER** to get back to the main menu. Press the **6** key to quit RANGER.

- To get to the table, press **STAT** and then press **ENTER** to select the first option ("Edit…").

List $L_1$ is used to store time information. List $L_2$ is used to store distance information. List $L_3$ is used to store velocity information.

17. Use the arrow keys to scroll down in list $L_1$ to find $t_o$ (see #10). What is the value in list $L_2$ to the right of the number for $t_o$? (Remember, $L_2$ stores distance information, so we will call this value the initial distance or $d_o$). ___________________

18. Use the arrow keys to scroll down list $L_1$ to find $t_f$ (see #12). What is the value in list $L_2$ to the right of the number for $t_f$? (We will call this value the final distance or $d_f$.) ___________________

- You are finished with your first trial run. You are going to repeat the experiment 9 more times, launching your model car from a variety of different points on the ramp in order to get a good range of initial velocities. You do not have to answer all the questions that you just finished for each trial, but you will need to fill out the following table of essential information.

19. Enter the information that you have already gathered in the first line of the table. The heading of each column of the table tells you the quantity to enter in that column as well as the question number you can refer back to help you find the data.

**Note: The fifth and eighth columns require you to subtract values found in the two previous columns.**

| Trial # | $v_o$ (#11) | $t_f$ (#12) | $t_o$ (#10) | $t_f - t_o$ | $d_f$ (#18) | $d_o$ (#17) | $d_f - d_o$ |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | | | | | | |
| 9 | | | | | | | |
| 10 | | | | | | | |

Well done! You are finished collecting the data you will need to explore the relationships between initial velocity, stopping time, and stopping distance!

Let’s crunch those numbers and see what we can find out…

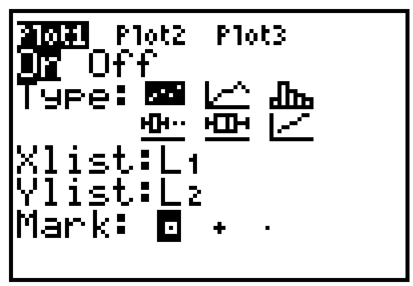
The numbers in the fifth column labeled “$t_f - t_o$” represent the amount of time it took for the car to go from the bottom of the ramp to a complete stop—the stopping time!
You are going to enter this information as well as the initial velocity information into the TI-83 to see if there is a relationship between these quantities.

- If you need to get back to the table of values, press the **STAT** key and then press **ENTER** to select the first option (“Edit…”). You will need to clear out the information found in these lists in order to enter new information. To clear out a list, first use the arrow keys to position the cursor on the title of the list that you want to clear out. Press **ENTER** then press **CLEAR**, and then press **ENTER** again.

- Enter the values from column 2 of the table titled “$v_o$” into list $L_1$. To enter a value, use the arrow keys to position the cursor in the first empty space in the list. Punch in the number using the number keys, and enter the number using the **ENTER** key.

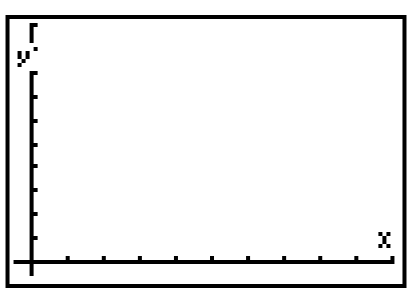
- Enter the values from column 5 of the table titled “$t_f - t_o$” into list $L_2$.

- Press the **2nd** key and then **Y=** to select “STAT PLOT” and then press the **ENTER** key. Use the arrow keys and the **ENTER** key to select among the options on each line. The only exceptions are the “Xlist” and “Ylist” options, which can be punched in by pressing the **2nd** key and then **1** for “$L_1$” or **2** for “$L_2$.” When finished, the screen should look like this:




- Press the **GRAPH** key to view the graph of your data points. If the window does not fit your points very well, press the **ZOOM** key and then **9** to select “ZoomStat.” This automatically adjusts the window size to fit your data points.

20. Draw what you see on your screen in the space below. Make sure that you draw your points large enough.



21. If this experiment were run under ideal conditions with more sophisticated equipment, these data points would all lie on a straight line. What are some reasons why your data might not be ideal? (You may use the back of this page if you need more room—you can probably think of many reasons!) ________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________

22. Using a ruler, draw a line on the picture above that you think fits your data the best.

23. Pick two of your data points that you think lie closest to this line. If you have many to choose from, pick two that are as far apart as possible. Press the **TRACE** key and use the ◀— and —▶ keys to get the coordinates of each of these points. Enter the coordinates of each point in the space below. You need to record only 3 significant digits.

    Point 1: ( _______, _______)    Point 2: ( _______, _______)

24. Calculate the slope of the line containing these two points. Use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \_\_\_\_\_\_\_\_\_\_\_\_$$

25. Use this value for *m* and one of your two points to write the equation of your line. Use the point-slope formula $y - y_1 = m(x - x_1)$.

______________________________

As expert scientists, you are now able to use this equation to predict the outcome of future experiments! The next time you are driving down a carpeted road in your model car and decide to coast to a stop, all you have to do is some quick calculation to figure out how long this will take. Let’s see how well your equation can predict the outcome of one final experiment.

- Run the original experiment one more time. Make sure that you are finished with all the data you have in the calculator before you begin, since the RANGER program will overwrite lists $L_1$, $L_2$, $L_3$, and $L_4$.

26. Use the **TRACE** key to find $t_o$, $t_f$, and $v_o$. Compute $t_f - t_o$. This is your actual value for the stopping time.

$v_o$: ________________ $t_f - t_o$: __________________

27. Plug your value for $v_o$ into your equation for $x$ and calculate $y$. This is the value for $t_f - t_o$ that your equation predicts. Enter this value here: ________________

28. Absolute error is defined as the absolute value of the difference between the actual value and the predicted value. Use this formula to compute your absolute error:

Absolute error = |Actual value – Predicted value| = ___________________

Looking at the absolute error by itself is not very helpful for judging how good of a prediction you came up with. To see why, consider that if you estimated the distance to the moon with an absolute error of 1 meter, that is an excellent prediction, but an absolute error of 1 meter would be horrible if you were estimating your height. That is why scientists often use relative error.

29. Relative error is found by dividing the absolute error by the actual value. Use this formula to compute your relative error:

$$\text{Relative error} = \frac{\textit{Absolute error}}{\textit{Actual value}} = \underline{\hspace{5cm}}$$

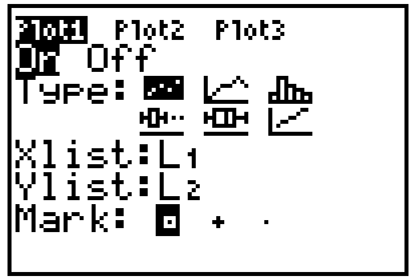
You have completed your investigation of the relationship between initial velocity and stopping time. Compare your guess at the beginning of the worksheet to the graph that you generated with the CBR and the TI-83. Now let's find out more about the relationship between initial velocity and stopping distance.

- Clear out lists $L_1$ and $L_2$ in the TI-83. If you need help doing this, remember that you can use **STAT ENTER** to choose the "Edit..." option, then use the arrow keys to position the cursor on the title of the list that you want to clear out. Press **ENTER**, then press **CLEAR**, and then press **ENTER** again.

- Enter the values from column 2 of the table titled "$v_o$ (#11)" into list $L_1$. To enter a value, use the arrow keys to position the cursor in the first empty space in the list. Punch in the number using the number keys, and enter the number using the **ENTER** key.

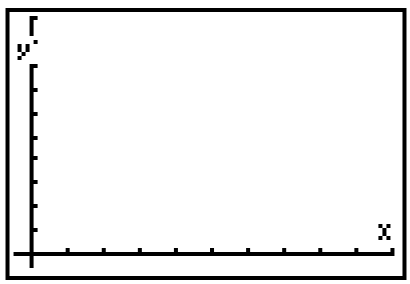
- Enter the values from column 8 of the table titled "$d_f - d_o$" into list $L_2$.

- Press the **2nd** key and then **Y=** to select "STAT PLOT" and then press the **ENTER** key. Use the arrow keys and the **ENTER** key to select among the options on each line. The only exceptions are the "Xlist" and "Ylist" options, which can be punched in by pressing the **2nd** key and then **1** for "$L_1$" or **2** for "$L_2$." When finished, the screen should look like this:




- Press the **GRAPH** key to view the graph of your data points. If the window does not fit your points very well, press the **ZOOM** key and then **9** to select "ZoomStat." This automatically adjusts the window size to fit your data points.

30. Draw what you see on your screen in the space below. Make sure that you draw your points large enough.




31. Is this relationship between initial velocity and stopping distance a straight line or "linear" relationship like velocity vs. time? ______________________

32. Imagine that two cars are driving down a straight road. One car is going twice as fast as the other car, and both cars hit the brakes at the same time until they both come to a complete stop. Circle the letter of the outcome that you think is likely to happen:

    A. The faster moving car will take the same distance to stop as the slower moving car.
    B. The faster moving car will take twice the distance to stop as the slower moving car.
    C. The faster moving car will take more distance to stop than the slower moving car, but less than twice the distance of the slower moving car.
    D. The faster moving car will take more than twice the distance to stop as the slower moving car.

Name:_________________________________ Date:_____________________ Period: _________

# Lab Activity #2:
# When will it stop? Where will it end?

## I. Objective:

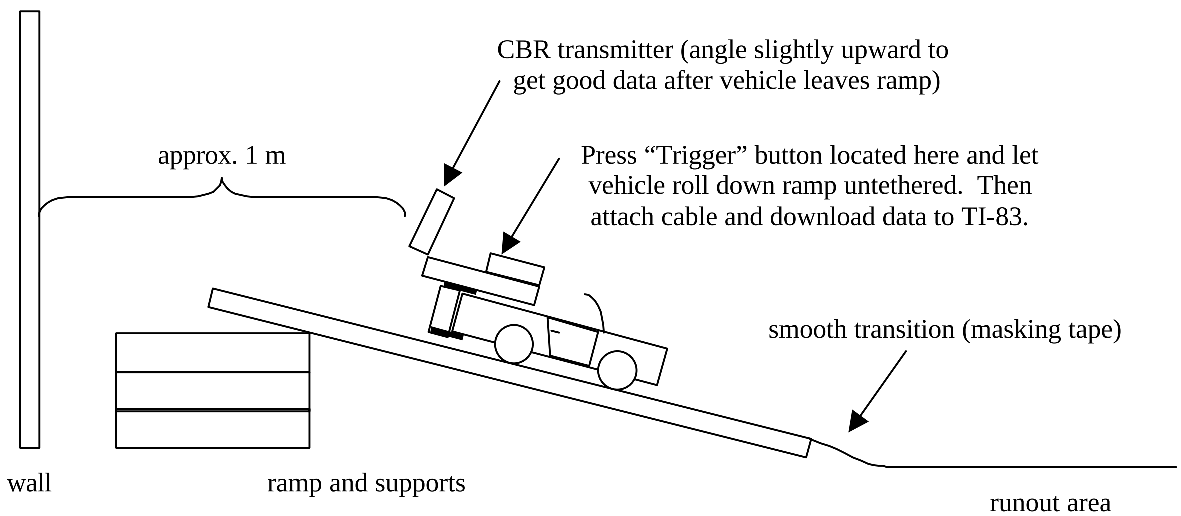
In this lab activity, you will be studying the relationships between a moving vehicle's velocity and its stopping time and stopping distance.

## II. Materials:

•TI-83 •CBR •Ramp •Vehicle large enough to mount a CBR

## III. Setting Up:

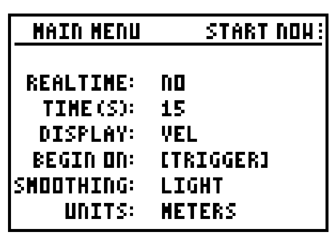
After setting up the ramp, begin the experiment by releasing the vehicle down the ramp as shown in the diagram below.




## IV. Procedure:

### *Part 1: Collecting the data*

Step 1: Set up your RANGER program to have the settings shown at right.

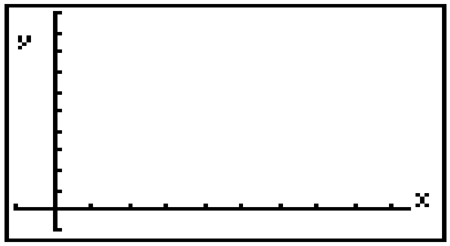



Step 2: With the CBR attached to the car, trigger the CBR and let the car accelerate down the ramp. Attach the TI-83 and collect the data.

Note:

- Vary the starting position of the car in order to gather varying velocities.
- For the sake of the experiment, the initial velocity will be read from the moment the car leaves the ramp.

## ***Part 2: Analyzing the Data and Graphs***




1. Sketch a graph of a random trial's plotted velocity:

2. What do the local extreme value(s) represent in this diagram?
______________________________________________________________________
______________________________________________________________________
______________________________________________________________________

3. Are there any inconsistencies in the graph? If so, what would account for these inconsistencies?
______________________________________________________________________
______________________________________________________________________

4. Using the Plot Menu, record the following data in the chart below by alternating from the DIST-TIME and VEL-TIME menus.


| Trial | Elapsed Time ($t_f - t_0$) | Elapsed Distance ($d_f - d_0$) | Initial Velocity ($v_0$) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| | | | |
| | | | |

## ***Part 3: Exploring the Relationships***

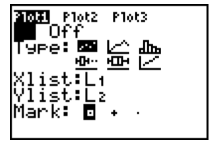
In the following steps, you will transfer data into the list function of the TI-83:

1. Clear any existing data in your lists by using the following keystrokes from the home screen:
**STAT 4 2$^{nd}$ 1 , 2$^{nd}$ 2 , 2$^{nd}$ 3, ENTER**

(Clear other lists in the same way if necessary.)

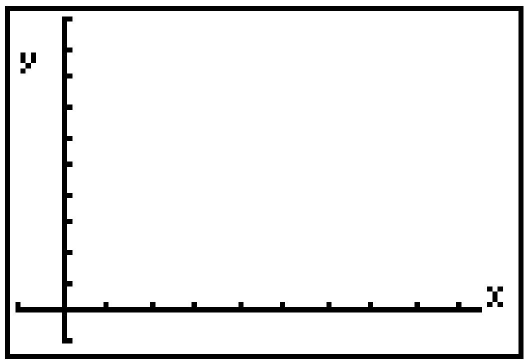
2. Transfer your findings of the initial velocity and elapsed distance data into lists $L_1$ and $L_2$ (or other suitable lists) on the TI-83 by pressing **STAT ENTER** and editing the lists.

3. Plot the relationship between velocity and stopping distance.
**2nd Y= ENTER**
Set the plot 1 menu settings as suggested at right.
Press **ZOOM 9** to plot the scatter plot.




4. Sketch a graph of the finding, labeling the independent and dependent variables:
*(In order to plot the relationship, you must have an independent variable (x-axis), which relies on nothing in order to be recorded, and a dependent variable (y-axis), which is affected in its variance by the independent variable.)*

Dependent variable: ______________




Independent variable: ________________

5. What type of relationship does this resemble? (Linear, power, quadratic, exponential, sinusoidal?)__________

## ***Part 4: Interpolation and Extrapolation***

In this part of the lab, you will try to predict the behavior of the vehicle using different methods: logical guessing, regression, and theoretical physics.

### *The Regression ("Best Fit") Approach:*

After plotting distance traveled as a function of velocity, use regression analysis to calculate a best-fit function:

**STAT ⟶4** (Use **4** for linear regression, **5** for quadratic, etc) **2nd 1 , 2nd 2 , VARS⟶1 1 ENTER**

Regression type used: _______________

Regression equation results: _______________________

### ***The Physics Approach:***

A branch of physics called kinematics, which studies motion of objects ignoring mass and force and assuming constant acceleration, has the formula for velocity given by

$$v_f^{\,2} = v_0^{\,2} + 2ad$$

where $v_f$ is the final velocity and $v_0$ is the initial velocity. Assuming $v_f$ is 0 m/s and solving for $d$ (stopping distance), the only variable left to calculate is acceleration, which will be assumed as constant. The computation of acceleration can be done by the following method:

$$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \frac{v_0 - v_f}{t_0 - t_f} = \underline{\hspace{2cm}}$$

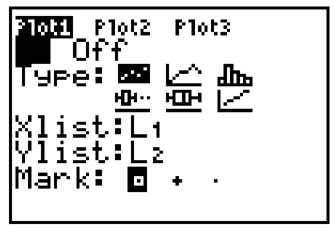
Plug in to the formula for distance and write your results:

$d = s(t) =$ ________________________

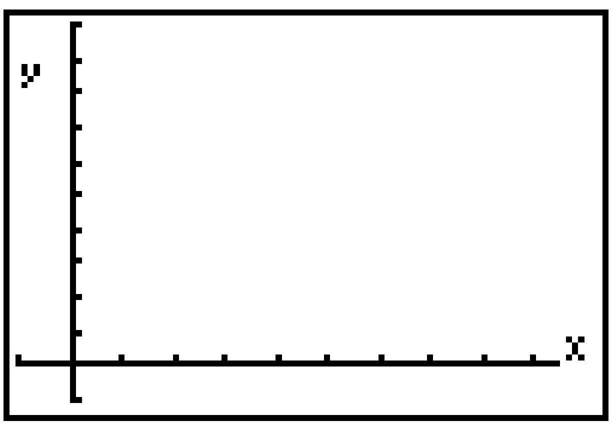
## V. Comparing Results: Extrapolation and Interpolation

Plot the following:

1. Plot 1 with following suggested setting: →
2. Regression equation: distance as a function of velocity (equation stored in $Y_1$)
3. Physics equation: distance as a function of velocity (equation stored in $Y_2$)




***Graph for Plotted Data points, Regression Equation, Physics Equation***




Now it is time to predict what will happen if the vehicle is traveling faster than any of your data collected up to this point. You will predict where the car will stop.

Release the car at the top of the ramp and record the final data:

| Initial Velocity ($v_o$) | Elapsed Time ($t_f - t_o$) | Your guess of stopping distance | Regression estimate of stopping distance | Physics estimate of stopping distance |
|---|---|---|---|---|
| | | | | |

Actual stopping distance using CBR's findings:_______

1. Which of the three methods was closest in determining the stopping distance of the vehicle? Why were the other two methods not as close?

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

2. What would you assume to be the most accurate method and why?

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

3. Study the two different methods of prediction: regression and physics. Both produce an equation for results past the points that you have collected. If the initial velocity is 0 m/s, what would the calculated stopping distance be? What could account for this error?

*regression*: ______meters *physics*: ______ meters

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

4. Looking at a state driver's manual, you will notice that one of the statistics commonly used is the braking distance recommended for cars in both normal and adverse driving conditions. Record the distance recommended to allow for stopping a moving vehicle at certain velocities.

__________________________________________________________

How well does this fit on your analysis graphs?

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

If there is discrepancy, what could account for this error?

________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

# TEACHER'S GUIDE

1. **<u>Development and Order of Procedures</u>**

   For best use of time, the apparatus should probably be set up in advance as shown below. Although students would learn a great deal by designing and setting up their own ramps and runout areas, doing this may take too much time.

   Regardless of the activity chosen (#1 or #2), begin by introducing the unit and relating it to the real world. Most students, especially sophomores and later, are very concerned with driving and, whether they admit it or not, are very interested in the distance needed to brake to a stop, and possibly also the time required. Explain that we will begin by looking at the time needed to come to a stop, and later we will examine the distance required. Questions for kicking things off: If you're driving 65 mph in a 55 mph zone, how much longer will it take you to come to a full stop than if you had been observing the speed limit? If you had a crash, would the police be able to tell from the length of your skid marks that you had been speeding?

   One effective way to present the unit is to frame it with a contest. Demonstrate that a variety of release points for the vehicle lead to a variety of ending positions, with (obviously) higher release points corresponding to longer runout distances. Show the students from what height you will release the vehicle and ask them to guess how far it will roll before braking to a stop; with any luck, the students' guesses will be varied enough and imprecise enough to give them the idea that predicting stopping distance accurately might be a bit of a challenge.

   After finishing the opening contest, organize students into groups if you have multiple calculators and/or CBRs. (Otherwise, choose students in rotation to be data collectors for the entire class.) Then proceed to the chosen activity (#1 or #2), and let students work at their own pace. It is important to monitor the early data collection to make sure that students are getting curves of reasonable shape. If the curves are too bumpy, remind them to turn smoothing on ("light" should be sufficient) as shown in the CBR setup instructions on their worksheets.

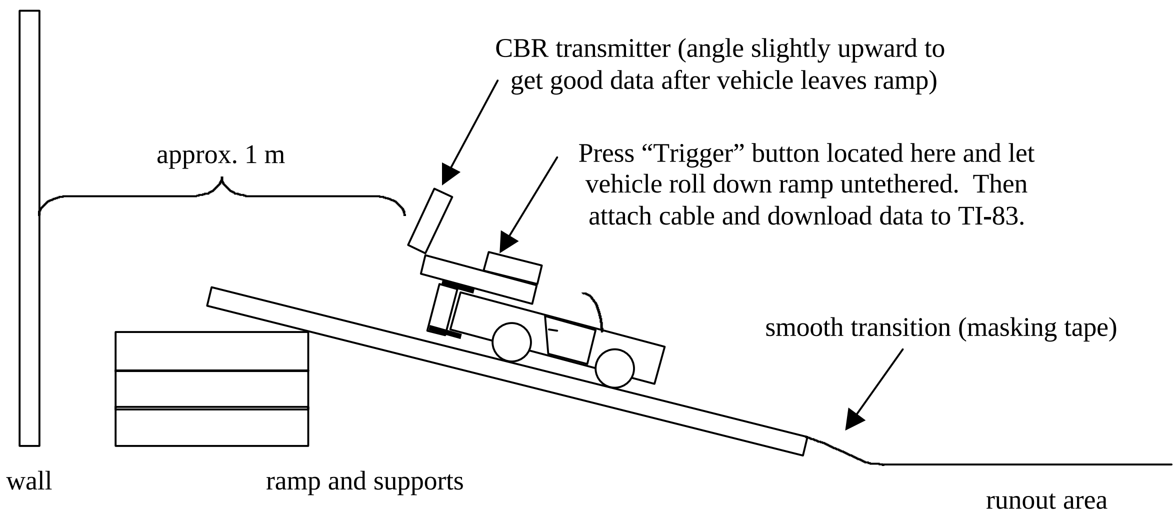
   At the end, or at least after a majority of the students have managed to achieve a predictor function, reassemble the students for a repeat of the guessing contest. Either release the vehicle from a position for which velocity has been calibrated, or release the vehicle and report its peak velocity to the students. Award a prize to the student (or group) that comes closest to guessing the true stopping distance.

   Note that the winning entry in the final contest may not necessarily be a student who plugs a value into a regression function, particularly if the contest $v_o$ is outside the domain of values seen during data collection. The winner may well be a student who applies the knowledge gained from the learning unit, plus a bit of special intuition about which direction the errors are tending. It would be appropriate to discuss why the winner was successful and ask students to explain what the implications are for empirical research. (Answer: There is no substitute for common sense. Mathematical models are helpful, but the real world frequently introduces complicating factors.)

2. **<u>Assessment</u>**

   For each activity, numbered questions walk the students through each step and provide opportunities for them to reflect upon what they have learned up to that point. Weighting of point values is left to the discretion of the teacher. In many cases, a single completion grade may be appropriate, with a bonus for students or groups who achieve accurate predictions in the final contest.

**3. Setup Diagram**




**4. Equipment List** (* denotes optional equipment):

At least one TI-83 or TI-83 PLUS calculator
At least one Texas Instruments Calculator-Based Ranger ("CBR")
Clamp for CBR
Cable for connecting CBR to calculator
Medium-sized toy car or wheeled physics vehicle
Metal or wooden ramp with smooth transition at end
Masking tape
Ramp supports
Level runout area (a carpeted floor is ideal)
Wall
* Post-It® flags or pushpins for marking students' guesses
* Meter stick or tape measure for mounting to ramp
* Pan balance or beam balance
* Weights

**5. Notes on Equipment** (listed in same order as above)

Calculators
Although this learning unit is written for a TI-83 or TI-83 PLUS calculator, you as the teacher could readily adapt these ideas to any other equipment combination that provides a motion detector, an interface, and software for manipulating and graphing the data. For example, an Apple Macintosh with a motion detector, a serial interface, and appropriate time/distance/velocity graphing software would work equally well.

Although one calculator will suffice, data gathering will go much more quickly if you have multiple calculators. Ideally, one student will use a calculator to analyze and report the data from a run, while another student rolls the vehicle and downloads the data for another run, and a third student builds scatter plots from the collected data. If everyone in the class has a calculator, organize students into groups of 3 or 4, with each group working from its own ramp.

**Note:** Even if the ramps differ significantly in their design, the data from multiple ramps can be pooled as long as the vehicles and runout surfaces are identical. See the similar notes under the "CBR" and "Ramp" headings below.

<u>CBR</u>
As noted above, this learning unit is written for the Texas Instruments CBR and TI-83 combination. However, you can readily adapt these ideas to other systems.

As shown in the diagram, be sure to angle the sonar transmitter slightly upward so that the waves will be bouncing off the wall, not the ramp. Otherwise, you may get false readings after the vehicle enters the runout area.

**Note:** If you have multiple CBRs as well as multiple calculators, the student groups can gather data quickly from one or more ramps by bolting a "fresh" CBR onto the vehicle while other students download and analyze the data from the previous CBR. This way, your students can gather more data in a shorter amount of time.

<u>Clamp</u>
The CBR needs to be firmly affixed to the vehicle. For some toy cars, duct tape may work better than the clamp provided with the CBR; however, using tape would eliminate the possibility of speeding up data collection with multiple CBRs (see note above).

It is true that you may get better results by mounting the CBR on a stationary surface and pointing the CBR at the vehicle going away from it. However, there are several advantages to initially mounting the CBR on the vehicle itself:

- Some vehicles, especially small toy cars with few flat surfaces, may not make good sonar targets. A large flat wall is an ideal target.
- The bumpiness of the moving vehicle adds "noise" to the data and provides an opportunity to ask the students for suggestions on how they would improve the apparatus and reduce noise. You can have them "discover" the fixed-CBR idea on their own and perhaps discuss the target-size tradeoff (see previous bullet) that would be caused.
- With multiple CBRs in the room, there is likely to be less interference if the transmitters are aimed at walls instead of at vehicles.
- The slight upward slant of the CBR (shown in diagram) is recommended in order to avoid false readings from sonar waves bouncing off the ramp. However, the upward slant also means that the sonar measurements are not necessarily made along a horizontal line. In fact, as the vehicle travels on the runout area, the measurements will be taken along the hypotenuse of a larger and larger triangle. Here is a good opportunity to discuss trigonometry briefly and ask the students how much the data will be distorted (see "Optional Extension Activities" below).
- The vehicle has a little more "personality" with the CBR attached to it. Students enjoy taking ownership of the instrumented cars, especially if each group can have a car of its own.

<u>Cable</u>
Use the long strain-relief cable provided with the CBR. Because of the frequent plugging and unplugging that will occur, it is important to use this type of cable, which has a straight connector instead of the angled connector on the shorter TI-83 link cable.

Vehicle

Because the rate of deceleration should be constant for this learning unit, the entire class must use a single vehicle if data are to be pooled.  If the groups use different types of toy cars or other vehicles, each group must gather data only for itself.  An exception would be if several highly similar, precision physics vehicles are available, in which case the data for all ramps and all vehicles could be pooled as long as the runout areas are identical.

Ramp

The design and construction of the ramp are not critical for the gathering of good data. However, if you would like to be able to make good predictions of stopping distance based on a calibration of the ramp, you should anchor the ramp firmly and make sure that the transition from ramp to runout area is as smooth as possible.

**Note:**  There is no need to have uniform slope and design of ramps.  The only thing the ramps are used for is to accelerate the vehicles to speed in a convenient, somewhat repeatable way; only the velocity at the bottom of the ramp (what we call $v_0$ in this learning unit) is relevant to the analysis that follows.  Even if the ramps differ significantly in their design, the data from multiple ramps can be pooled as long as the vehicles and runout surfaces are identical.  See the similar note under the "CBR" heading above.

Masking Tape

Use a generous amount of masking tape, a stiff binder, or other suitable material to make the transition from ramp to runout area as smooth as possible.

Ramp Supports

Physics books or obsolete software manuals make ideal ramp supports.

Runout Area

The runout area must be uniform since it must decelerate the vehicle at a constant rate.  Also, because of CBR range limitations, the runout area should bring the vehicle to a full stop in the space of a few meters.  A carpeted floor is ideal.

Be sure that the runout area is long enough that the vehicle will not crash into any objects.  If students occasionally run a vehicle into an obstruction, ask them what they think they should do with the data.  (Answer:  Discard that data point and perform a new run, possibly from a lower start position.)

Wall

The wall should be flat and uniformly textured.  Do not use a wall that has a few posters that will be hit by the transmitter from some angles but not from others.

Flags or Pushpins

Post-It® brand flags are ideal but are fairly expensive (about 3 cents each).  However, they are safer than pushpins for marking student guesses.  Masking tape also works fine but is not as colorful.

Meter Stick or Tape Measure
If you wish to calibrate the ramp for the contest (i.e., to mark the $v_0$ corresponding to various starting positions), a meter stick or tape measure attached to the ramp is very convenient. However, pencil markings or masking tape markings will also work.

Pan Balance, Beam Balance, Weights
If you wish to pursue the optional extension activities involving varying the mass of the vehicle (see below), an accurate balance will be essential.

## 6. Optional Extension Activities

### A. Engineering-type Questions to Pose to Your Students

**(1) What would happen if the CBR were mounted on the car facing forward toward a wall on the other side of the room (instead of backward as shown in the diagram)? Would this cause a problem in interpreting the data?**

**Answers:** In this case, since the distance between CBR and target (wall) is always decreasing, *ds* (the instantaneous change of distance) is always negative. Therefore, *v*, which is defined to be $\frac{ds}{dt}$, must also be negative. This causes no problem in our analysis as long as we remember that the usual real-world interpretation of velocity of the vehicle as it comes off the ramp is positive. For convenience, we would probably record positive velocity values throughout. This is an example in which a sign difference is essentially trivial (unlike, for example, a situation in which somebody miscalculates the sign of a second derivative and treats a minimum as a maximum or vice versa).

**(2) What sort of device does a state trooper parked at a speed trap use to determine the speed of passing cars? Does it matter whether the cars are approaching or receding from the trooper? What is the easiest way of resolving this issue?**

**Answers:** State troopers and other police officers use a device that is very similar to the CBR in principle, except that the police equipment is built to higher precision standards, has a much greater range, and uses radio frequency (radar) or laser light waves ("lidar") instead of sound waves. It does matter whether vehicles are approaching or receding, since the closing velocity will be negative if the vehicles are coming toward the trooper, and positive if the trooper aims at the rear of vehicles as they pass by. The easiest way of resolving this issue, though, is simply to have the radar or lidar gun display the *absolute value* of the computed velocity.

**(3) What would be some ways of reducing "noise" in the experimental data?**

**Example answers (many more are possible):**

- Try keeping the CBR stationary (remember the tradeoff, though, since the vehicle will now need a large flat area to serve as a target).
- Use low-pile indoor/outdoor carpet so that the deceleration surface is more uniform.
- Use a low-friction vehicle attached by fishing line to a mass on a pulley to provide the braking force.
- Use a specially fabricated metal ramp with a negligible transition bump, or better yet, a ramp and runout area that are a single unit.

**(4) Why did your teacher ask you to angle the CBR transmitter slightly upward? Does this cause any distortion of the data? Explain.**

**Answers:** When setting up the apparatus, you can reduce the chance of getting a false reading from the ramp by pointing the CBR slightly upward. When the vehicle enters the runout area, it will be measuring distance along the hypotenuse of an ever-growing triangle, *not* the straight-line distance to the wall. However, the length of this hypotenuse is virtually identical to the straight-line distance. The distortion of the data will be negligible as long as the vehicle start point is reasonably far from the wall (for example, 1 m as suggested in the diagram).

**(5) If your vehicle crashes into a wall or other object, is there a way that you can use the data resulting from that trial? Explain.**

**Answer:** You must discard the data from such a trial, since there is no way to make it comparable to the other trials, in which deceleration is (hopefully) uniform.

## B. Ideas for Additional Research

**(1) Compute the value of *g* (the gravitational constant).**
By carefully measuring the angle of the ramp, or by using trigonometry to compute the angle of the ramp, students can resolve the weight of the vehicle into normal and accelerating force components. Then, for a carefully measured start position, the velocity at the bottom of the ramp can be measured repeatedly, averaged, and utilized to compute the value of $g$ indirectly. This is really more of a physics exercise than a mathematics exercise, but some students may find it interesting. Point out to students that the value of $g$ varies over the earth's surface and that underground features (pools of oil, for example) can sometimes be located through careful analysis of gravitational data generated by highly sensitive instruments.

**(2) Analyze the effect of vehicle mass on stopping distance.**
Students should understand intuitively that a heavier vehicle will take longer to stop from a given starting velocity (both in time and distance) than a lighter one, provided that the braking force remains constant. By measuring the mass of the loaded vehicle and changing it to several different values, students should be able to generate a family of curves, either for stopping time vs. velocity or for stopping distance vs. velocity, in order to gain some insight into this phenomenon and to conjecture on the nature of the relationship between mass and stopping distance for a fixed initial velocity ($v_o$).

A major complicating factor with such an experiment is that there is no simple way to keep the braking force constant. A heavier vehicle will tend to dig more deeply into the carpet, thus generating more friction, and it will have more axle-related friction as well, especially if the vehicle is a simple toy without ball bearings. All of these considerations may overwhelm the expected increase in stopping distance and lead to false conclusions. (Ideally, stopping distance should increase as momentum, computed by $mv$, increases.)

One way to extend the learning unit in this situation, without redesigning the apparatus, would be to perform some tests in advance and choose weights such that the heavier vehicle stops in a shorter time. Then ask the students if this agrees with their intuition (hopefully, it will not) and ask them to explain what factors are causing the experiment to fail.